A short Mathematical Analysis of a Fern

by

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(With 4 figures in text.)

Abstract

A mathematical analysis of a fern was done based on measurements with a caliper and a ruler. The length of the main branch, the distance to each point of branching and of each leaf is determined and their values are plotted in a x-y diagram. It is revealed that the quotient of the lengths of the branch to the distance of branching is dependent approximately to the square root of the quotient of leaf length to leaf number.

1 Process of Measurement

The fern studied grows in our garden and is shown in Figure 1.
Figure 2 shows the prepared branch (2) of the fern with its specific leaves (1). The branch (2) was cut at the lowest possible end where it originates from the root and stretched as much as possible during measurement. There are 18×2 leaves (1), which where able to be measured. Measurement of the fresh cut branch was done in a pragmatic way, i.e. without being lost in details of measuring to half a millimeter or so. Furthermore measurement was done naively without knowing any details of the result and also without intentional thoughts. Figure 3 shows the notes of the measurement.

The whole length $\mathcal{L}$ of the main branch was determined to be 605 mm. As mentioned before, there are 18 determinable knots $\lambda_i (i = 1, 2, 3, \ldots 18)$ (3), where the appropriate leaves originate. λ_i obviously equals the leaf number. The length of a leaf starting from the knot, i.e. the point of branching is denoted as ℓ. At each knot, for both leaves ℓ was measured and the mean was calculated from the two values.
The corresponding length $\mathfrak{L}$ from the beginning of the branch where it was cut to the appropriate knot was also determined for each pair of leaves.

Figure 1: Fern studied

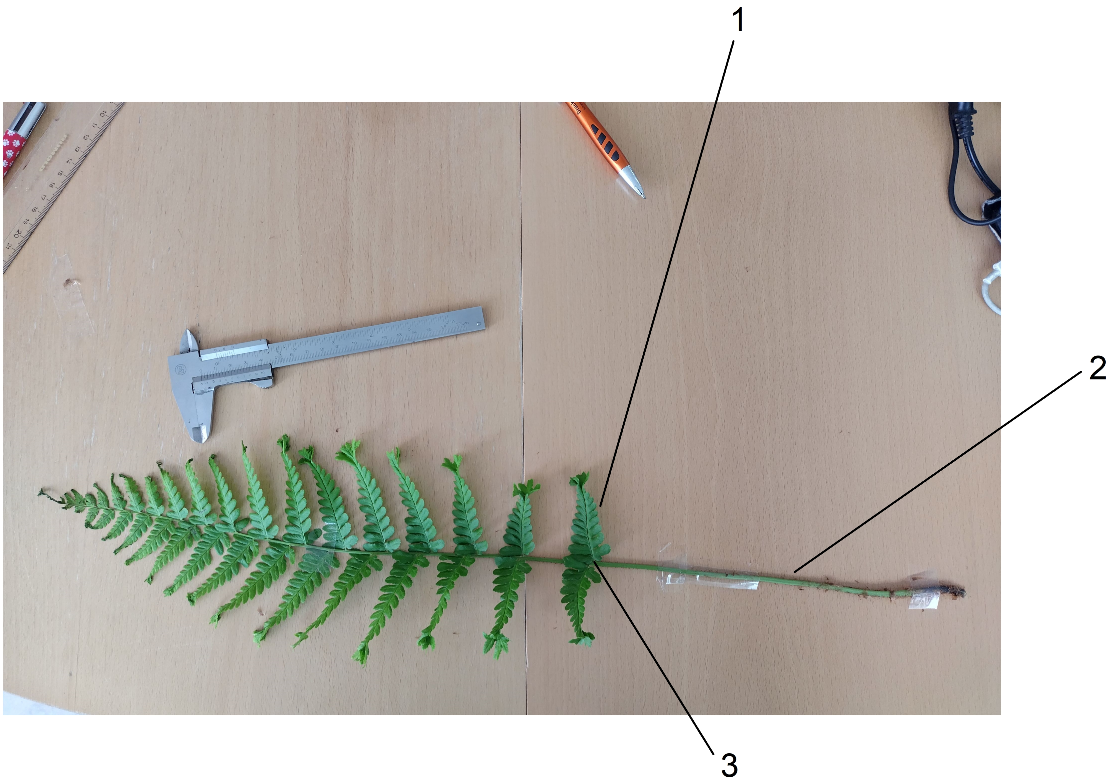
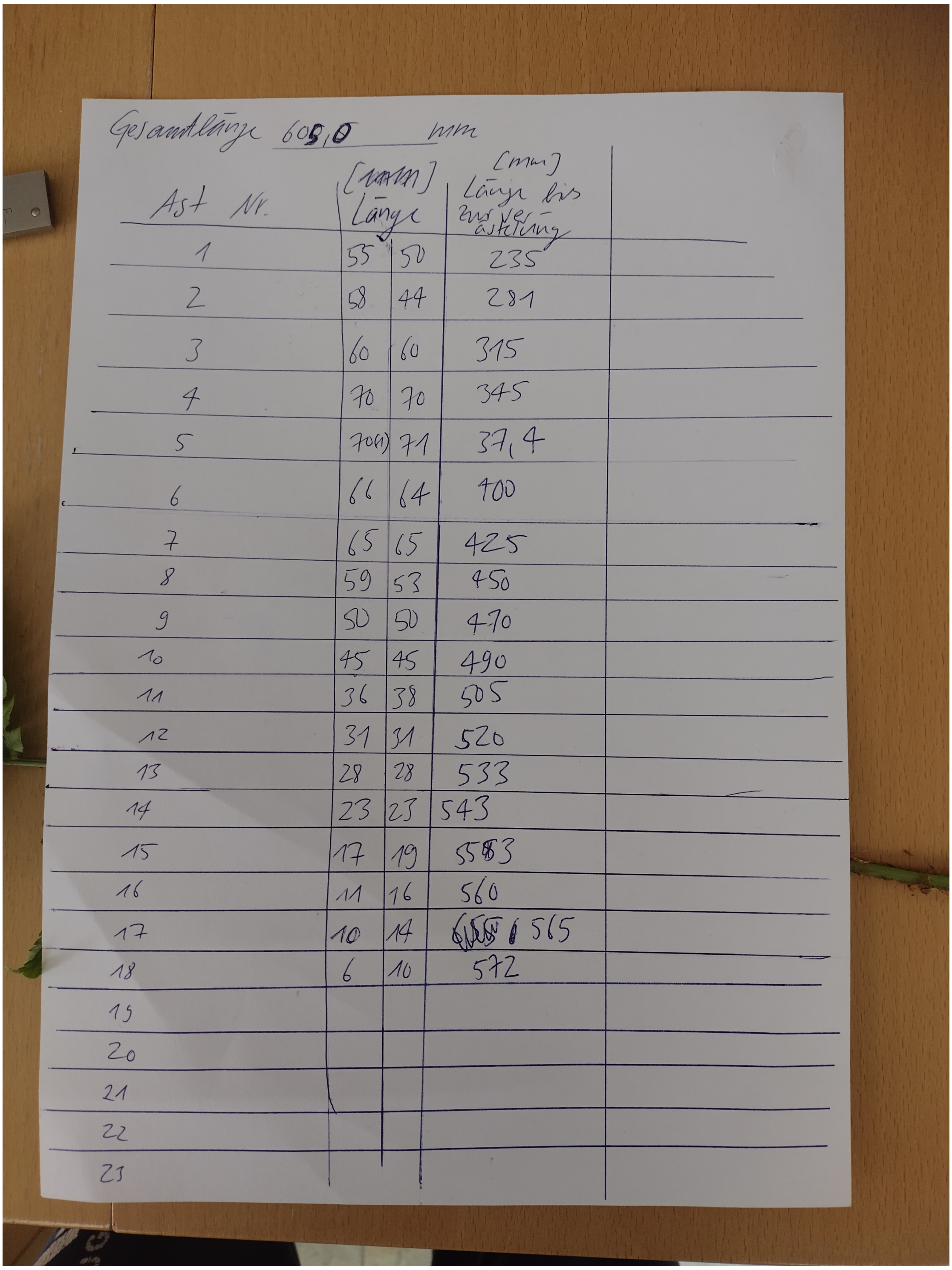


Figure 2: Prepared fern on table with measuring instruments

1 leaf
2 branch
3 knot, location of branching



Gesamtlänge 605,0 mm

Ast Nr.	[mm] Länge		[mm] Länge bis zur Verästelung
1	55	50	235
2	58	44	281
3	60	60	315
4	70	70	345
5	70(1)	71	37,4
6	66	64	400
7	65	65	425
8	59	53	450
9	50	50	470
10	45	45	490
11	36	38	505
12	31	31	520
13	28	28	533
14	23	23	543
15	17	19	553
16	11	16	560
17	10	14	565
18	6	10	572
19			
20			
21			
22			
23			

Figure 3: Notes of measurement

2 Mathematical Evaluation

Given the measured values, the following two quotients were determined:

$$\left.\begin{aligned} x &= \frac{\ell}{\lambda_i} \\ y &= \frac{\mathcal{L} - \mathfrak{L}}{\mathcal{L}}\,. \end{aligned}\right\} \tag{1}$$

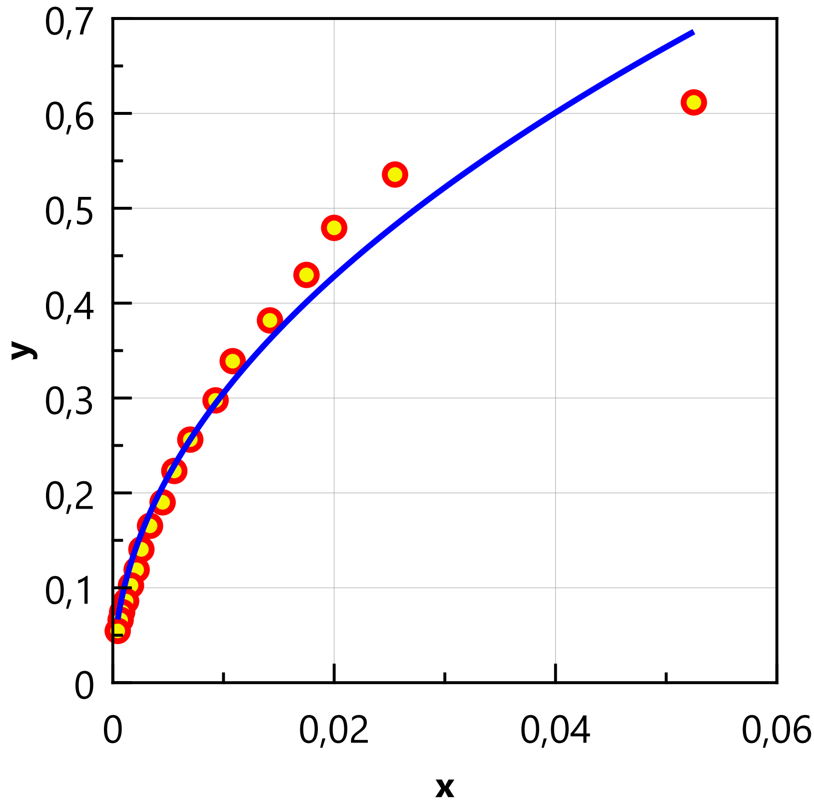


Figure 4: Plot of measured values of a fern, for **x** and **y** see Equation 1

These values x and y were plotted against each other, the plot is shown in Figure 4. For evaluation all values were recalculated to metre. If a regression analysis is done with LabPlot, the following relation

$$\frac{\ell}{\lambda_i} = 2.895 \left(\frac{\mathcal{L} - \mathfrak{L}}{\mathcal{L}}\right)^{0.489} \tag{2}$$

is obtained with regression values shown in Table 1:

sum of quadratic errors (χ^2)	0,0159246
remaining mean square (χ^2/dof)	0,00099529
square root of mean quadratic error (RMSD/SD)	0,0315482
coefficient of determination (R^2)	0,969184
corrected coefficient of determination (R^{-2})	0,965075
χ^2-Test ($P > \chi^2$)	1
F-Test	503
P ¿ F	0
mean absolute error (MAE)	0,0235099
Akaik's information criteria (AIC)	-69,5
Bayesian information criteria (BIC)	-66,8

Table 1: Goodness of adaption for plot shown in Figure 4

Equation 2 may be approximated with the formula

$$\frac{\ell}{\lambda_i} = \text{ const. } \cdot \sqrt{\frac{\mathcal{L} - \mathfrak{L}}{\mathcal{L}}}\,. \tag{3}$$